Study of the movement of microparticles at the water surface in a structure of Faraday waves

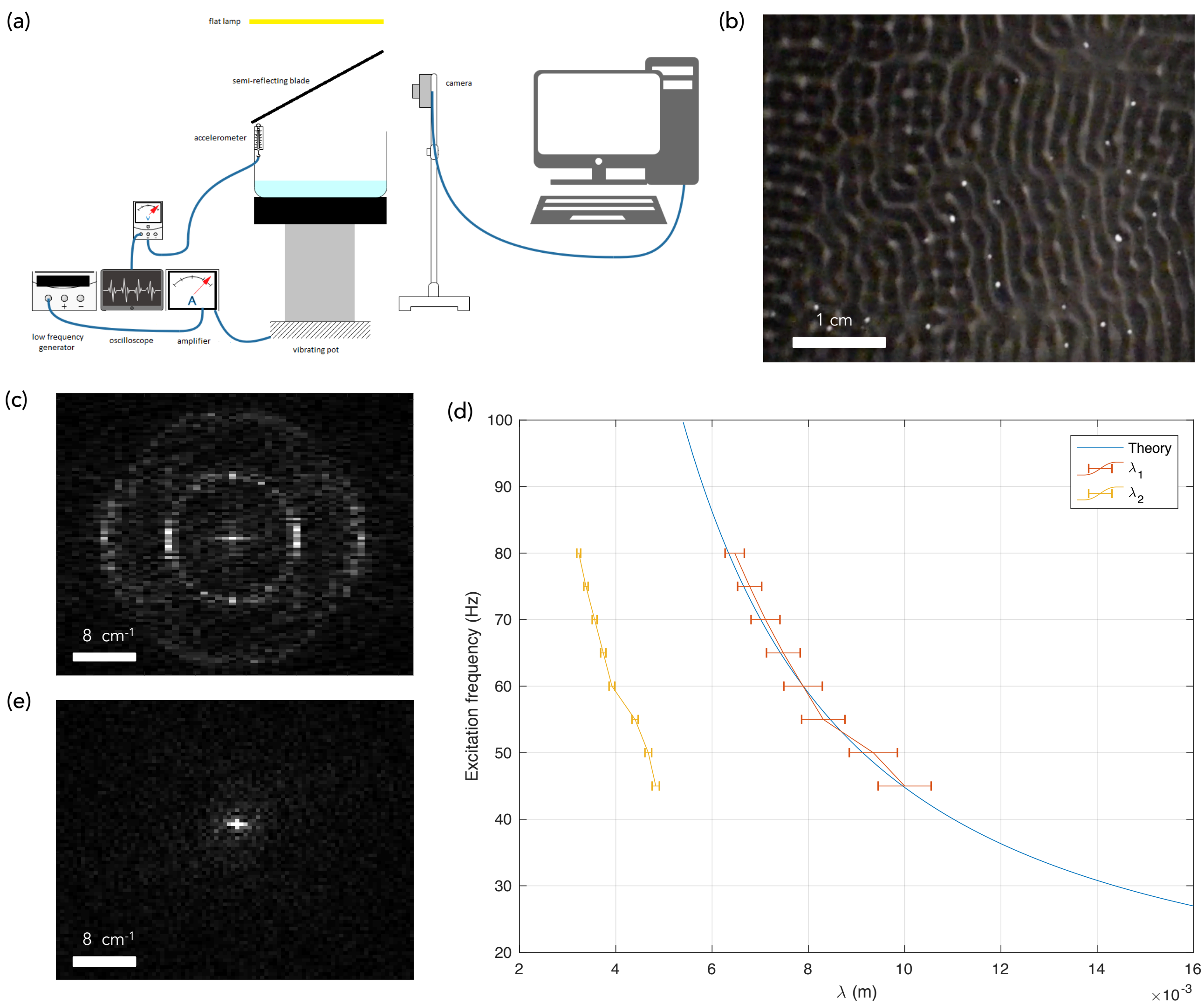


Figure 1 – Spatial wavelengths

(a) The experimental set up is composed of a vibrating pot connected to a low frequency generator by means of an amplifier. On this vibrating pot is fixed a tank (20x20 cm^2) filled with water (500 mL). The set up is illuminated from above by a flat lamp and the image of the tank is redirected to the camera thanks to a semi-reflective blade. On the one hand, the vertical tank acceleration is measured thanks to an accelererometer (9,809 mV/(m/s^2)) connected to an oscilloscope. On the other hand, the camera is directly connected to the computer in order to visualize in live the pictures and to capture them easily. **(b)** Picture (5x4 cm^2) of the water surface of the tank for an excitation frequency of 60 Hz and a vertical tank acceleration of 357.6 m/s^2 showing the mobile particles (50-100 µm) on the surface and the structure of surface waves. It reveals that particles are in the cavities and that the surface structure arranges around particular wavelengths. **(c)** The 2D Fast Fourier Transform of pictures – converted to binary image with a threshold – of the water surface without particles reveals the organization of the structure of the surface waves, in an isotropic way, around two wavelengths only depending on the frequency of excitation and not on the amplitude of excitation. The experimental data **(d)** are coherent with the theoretical curve for surface waves with a surface tension estimated at 55 mN/m. Note that a two times shorter wavelength is apparent since the 2D FFT detects mainly the distance between two « wave peaks » or two « wave hollows » which corresponds to half a wavelength. Nonetheless, figure **(e)** shows that the 2D Fast Fourier Transform of an image of the particle trajectories captured during a total time of a few thousand seconds does not reveal any particular wavelength. This observation illustrates the fact that the structure of the surface waves is moving.

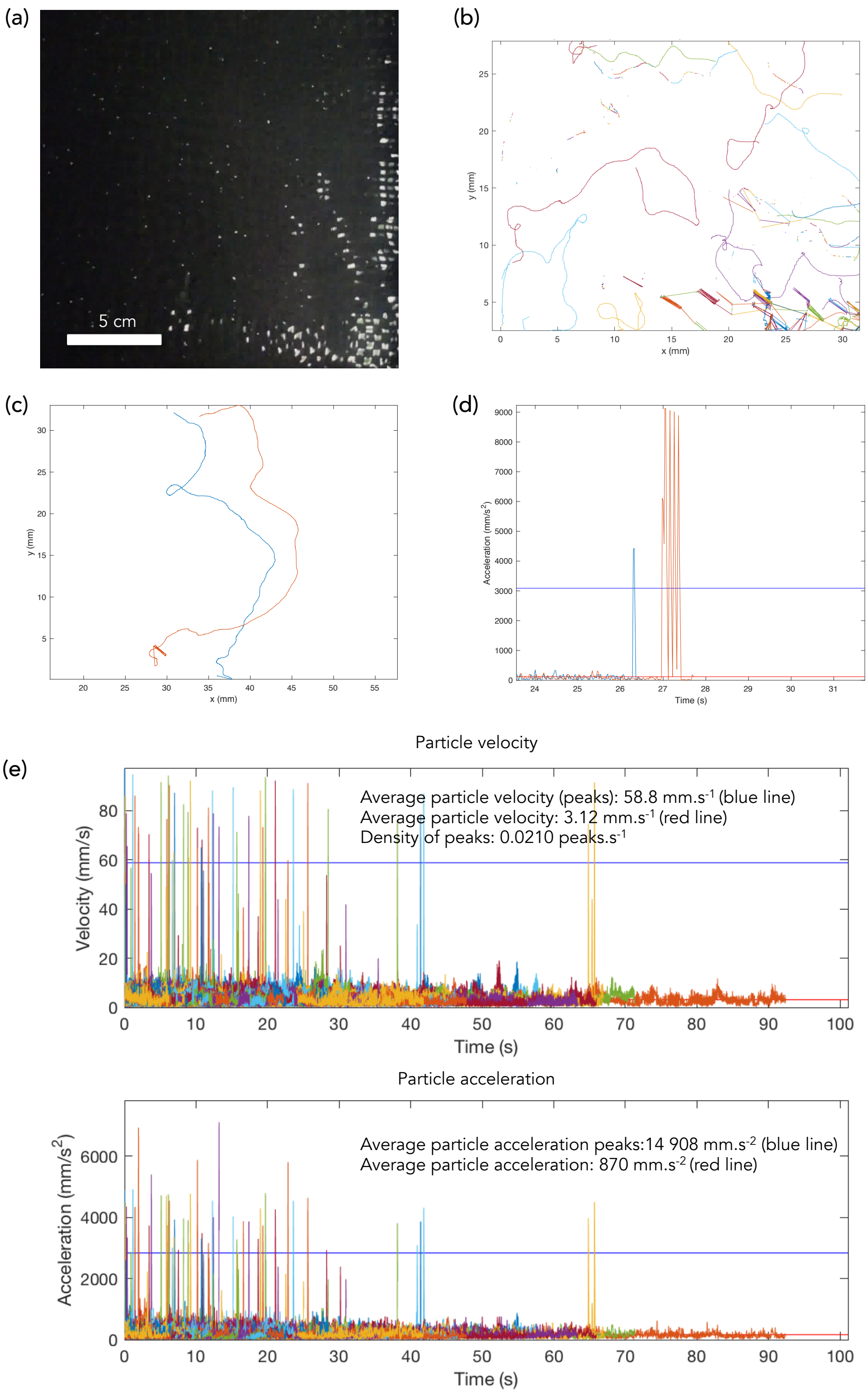
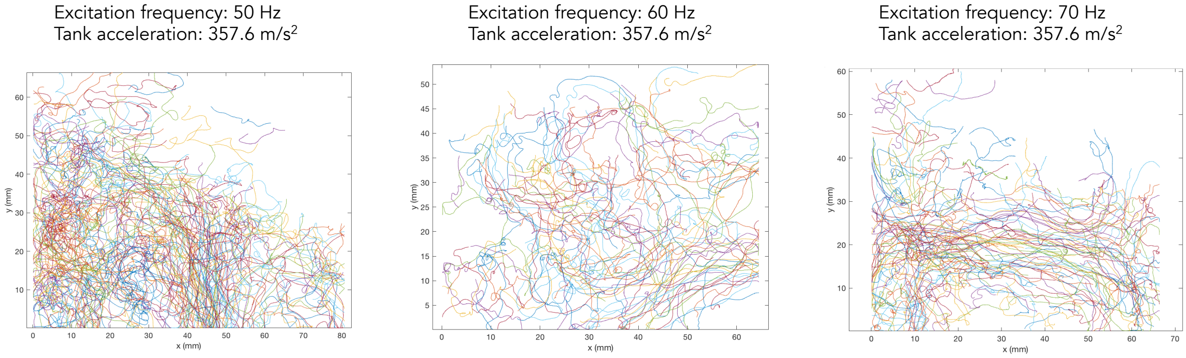


Figure 2 – Suppression of reflections in data processing on MATLAB

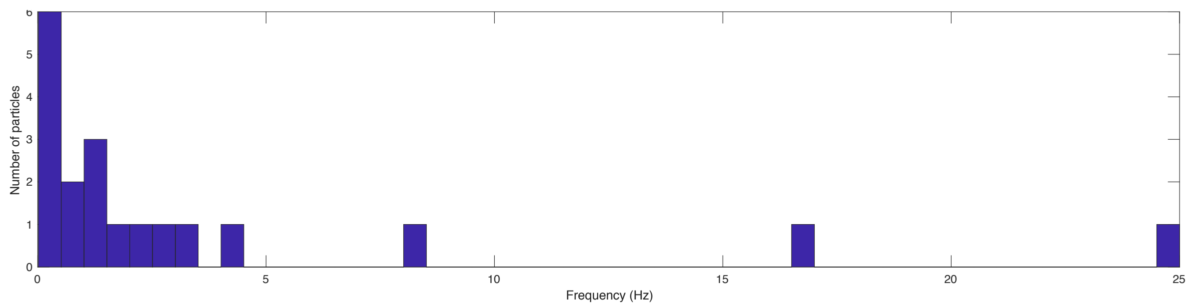
Studied case : (a), (b), (c), (d) excitation frequency of 60 Hz and vertical tank acceleration of 357.6 m/s^2 ; (e) excitation frequency of 50 Hz and vertical tank acceleration of 415.2 m/s^2

(a) Picture of the total tank surface illustrating the presence of reflections more or less near the edges of the tank linked to the combination of the lighting and the irregularity of the water surface. Note that the mobility of the surface waves induces a local mobility of the reflections. (b) Image of the particle trajectories for a video of 30 s processed with the plugin Mosaic – Particle Tracker 2D on ImageJ and with MATLAB without any reflection treatment. Due to reflections and to the relatively small memory capacity of ImageJ a compromise was made between the size of the particles considered by the plugin and the number of pictures processed in a row. It implies the difficulty for the plugin to correlate the same particle between two pictures sometimes (see small points on the image) and the transcription of the local mobility of the reflections (see scratches on the bottom right). (c) and (d) The application of a first reflection treatment based on the removal of very localized trajectories and low duration of tracking has eliminated the majority of the reflections and uncorrelated particles. Nonetheless, the plugin can make occasional mistakes making a correlation between a particle and a reflection (see orange trajectory on figure (c)). It can be easily identified looking at the acceleration of this particle which reveals many high peaks in a short lapse of time as opposed to the single peak of acceleration for a regular particle (see the difference between the orange and the blue curves on figure (d)). (e) The application of a second reflection treatment based on the elimination of the trajectories with many high peaks in a short lapse of time reveals the sporadic phenomenon of particle acceleration (see each colour corresponds to a particle) linked to the mobility of the surface structure for the treatment of a video of 168 s.

(a)



(b)



(c)

Excitation frequency (Hz)	Minimum deviation from a stationary frequency (Hz)	Tank acceleration (m/s²)	Comments on trajectories	Number of particles followed	Total time of tracking (s)	Average particle velocity (mm/s)	Average particle velocity (peaks) (mm/s)	Density of peaks (peaks/s)	Average particle acceleration (mm/s²)	Average particle acceleration (peaks) (mm/s²)
50	1.27	357.6	66x82mm² - slight tendency to vertical circulation	321	5294	3.11	57.81	0.0210	865	13916
50	1.27	415.2	70x95mm² - vertical circulation	338	5572	3.12	58.82	0.0210	870	14908
60	0.245	357.6	53x66mm² - homogeneous occupation	134	2991	2.17	58.00	0.0053	619	12578
60	0.245	415.2	61x84mm² - vertical circulation	226	4309	2.42	57.31	0.0149	804	13778
60	0.245	510.4	55x63mm² - homogeneous circulation	70	1135	2.84	56.74	0.0353	775	12666
70	1.184	357.6	60x66mm² - horizontal circulation	222	5590	1.77	49.45	0.0095	585	12144

(d)

$$Average\ particle\ velocity\ (peaks)\ \alpha\ \sqrt{h_{wave}}$$

$$Particle\ mobility = C^{st}.(\sin\left(\frac{L_{tank}.\pi}{\lambda}\right))^{C^{st}}$$

$$Average\ speed\ (or\ density\ of\ peaks) = (Particle\ mobility).(C^{st} - C^{st}.e^{C^{st}.\left(\frac{vertical\ tank\ acceleration}{h_{water}.f_{excitation}^{\ 2}}\right)^{Cst}})$$

Figure 3 – General trends around the results

(a) Three images showing the particle trajectories for three different captures of 168 s with different frequencies and the same amplitude. A tendency to a vertical circulation at 50 Hz and a horizontal one at 70 Hz are observed while the occupation of the space is almost homogeneous at 60 Hz. We cannot explain this observation. Note that due to the increasing presence of reflections with the increase of the excitation frequency, the space considered to treat the videos is different for each image. **(b)** For an excitation frequency of 50 Hz and a vertical tank acceleration of 415 m/s² a tendency to a decrease in proportion to 1/frequency of the spectrum appears - which is typical of a sporadic phenomenon. Nonetheless, we have a lack of data to confirm this tendency since only hundreds of particles were followed in each video which is not enough for such statistics. **(c)** Chart summarizing the data on the particles for different excitation frequencies and vertical tank accelerations. We have little data because of the impossibility to treat some videos because of reflections and their impact on the video processing time on ImageJ and MATLAB. However, from our observations and our data, we can draw a few tendencies **(d)**. First, we think that the average particle velocity (peaks) is linked to the wave height almost like if the particle was in free falling. Second, it seems that we could define a mobility for our particles which would be directly linked to the mobility of the surface structure depending on the wavelengths observed at the water surface and the tank dimensions. Thirdly, we can draw from the chart the same tendency for the average velocity and the density of peaks with different constants. In a nutshell, the important variables in this experiment are the vertical tank acceleration, the excitation frequency and the tank dimensions. Note that the mathematical trends are only applicable for a vertical tank acceleration above the critical acceleration to see Faraday waves on the surface and below the critical acceleration to see waves braking on the surface.